Measuring Asymmetric Persistence and Interaction
Effects of Media Exposures Across Platforms

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Abstract

In this paper, we explicitly model and estimate the effect of paid, owned and earned media exposures, including television, online banner ad, and Facebook exposures, on purchase behavior at the household level. We use an advertising goodwill model, allowing for asymmetric decay rates for platform-specific goodwill stocks, and incorporate two levels of interactions. First, we include interaction effects between these goodwill stocks in the consumer utility function. Second, we allow for interactions in exposures across platforms in the goodwill production functions. We use hierarchical Bayesian methods to estimate the model, incorporating platform-specific models of exposures to control for endogeneity due to firms’ ability to set aggregate levels of advertising as a function of expected demand, as well as their ability to target specific types of consumers. Our single source data allow us to assess both the short-term and long-term marginal contributions of paid, owned and earned media on sales at the consumer level; we find no meaningful interactions in the consumer utility function, but we do find a positive interaction between TV and online exposures in the creation of goodwill. On average, Facebook exposures have an insignificant effect on purchases although there is considerable heterogeneity in its effect.
1 Introduction

Although recognized as a primary concern for marketers, interaction effects across media platforms are high understudied. One reason for this is the lack of quality, single source data which would allow for the estimation of interactions effect at the consumer level. This problem is exacerbated in the realm of earned media exposures. Facebook Inc. has built a $3 billion-a-year advertising business by convincing marketers to buy new forms of advertising designed to create buzz around their brands. However, a recent Wall Street Journal article highlights advertisers’ doubts over whether they are getting their money’s worth from advertising on Facebook. What is the value of a “Like”? And how might earned media exposures interact with exposures on more traditional advertising platforms in ultimately driving consumer purchases?

“Attribution” is a recent trending buzzword in marketing used to describe how much of a sale can be attributed to a specific exposure. If TV advertising drives a consumer to a website and she ultimately purchase the product after clicking an online ad, is TV being given its due credit for driving the sale? Theory is quiet in regards to whether we can expect exposures across channels to be either compliments or substitutes - the question is inherently an empirical one. We allow for platform-specific goodwill stocks and their interactions in consumer utility, and we depart from the literature by allowing interaction effects not only in the effect of exposures on consumer utility (and actual purchase behavior) but also in the generation of the platform-specific goodwill stocks.

\[1 \text{See The Big Doubt Over Facebook.}\]
which is where we believe a priori that positive interaction effects are likely to occur, through increased brand awareness or salience, increased attention, etc. We further allow for platform-specific goodwill decay parameters to capture differences in long term effects of exposures across platforms.

Methodologically, we contribute to the literature by providing a framework to estimate advertising interaction effects in the presence of platform-specific decay andsatiation effects, controlling for endogenous exposures. Exposures are endogenous for two main reasons. First, the level of advertising by a brand may be a function of unobserved (to the econometrician) demand shocks. Second, the advertising might be targeted to particular consumers - this is true for all platforms, but especially true for online exposures. We expand the method used by Manchanda et al. (2004), who model firms’ ability to target based on individual-level demand response parameters to include their ability to set advertising levels as function of the unobserved demand shock, accounting for both sources of heterogeneity. We simultaneously estimate a reduced form model of individual-level media exposures using exposure data from an unrelated product category (or categories) to shift the level of exposures, and we provide simulations to demonstrate the method’s performance. We substantively contribute to the literature by providing estimates of platform-specific effectiveness of exposures in both the short and long run, and interaction effects in both the generation of goodwill stocks and of the goodwill stocks on consumer utility, using actual purchase data.

The rest of the paper is organized as follows. In the next section, we provide the con-
ceptual background for our research question and discuss the relevant literature. Section 3 outlines our model, and section 4 describes our estimation method and provides simulation results demonstrating its efficacy in recovering the model primitives. In section 5, we describe the data, and the results are discussed in Section 6. We provide our concluding remarks in Section 7.

2 Literature

Traditional advertising response models relate advertising levels to product sales, incorporating phenomenon such as the S-shaped (or concave) response curve, effects of competitors’ advertising, and time varying effectiveness; for a review of the classic literature, see Little (1979). Long term effects of advertising have been captured using a stock of advertising goodwill that decays over time but is replenished through more advertising according to some goodwill production function which reflects the effectiveness of advertising Nerlove and Arrow (1962); Ephron (2002). The Nerlove-Arrow model is given by:

$$\frac{dG(t)}{dt} = qg(A(t)) - \delta G(t),$$  

(1)

where $q$ is advertising effectiveness, which decreases with the level of current advertising, and the authors use the linear production function $g(A) = A$, which together lead to the concave production of new goodwill; $\delta$ is the goodwill decay rate.

Although there is an extensive literature on advertising response and effectiveness,
there is a lack of research that estimates interaction effects between different advertising channels; even in models with disaggregate data on household-level exposures, advertising is usually aggregated over different channels. The integrated marketing communications literature has shown the importance of interactions between different marketing channels (Schultz and Kitchen 1997; Leclerc and Little 1997; Naik et al. 2005; Vakratas and Ma 2005; Smith et al. 2006), but these interactions are often not included in models that account for wearout effects and/or long term effects of advertising on sales through an accumulated goodwill stock.

One exception is Bruce et al. (2012) who extend the dynamic linear model of Naik et al. (1998), allowing advertising and word of mouth effectiveness in augmenting goodwill (given by $q$ in the Nerlove-Arrow model) to vary according to a stylized model, which then drives movie demand. However, the authors do not in fact have measures of word-of-mouth, instead using movie ratings as a proxy; one potential criticism of these results is that the movie ratings will be highly correlated with movie quality which may be correlated with advertising, leading to an endogeneity concern. The authors find evidence of both wearin and wearout for advertising (for theater and video demand, respectively) and a positive interaction effect with the valence of movie ratings. Bass et al. (2010) use a

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1 Naik et al. (1998) use a goodwill accumulation model with a stylized dynamic linear model (DLM) for advertising effectiveness, incorporating repetition and copy wearout (or wearin) and forgetting in advertising effectiveness as a multiplier for the (linear) goodwill production function. Repetition wearout refers to declining effectiveness of (current period) advertising as a function of advertising frequency, and copy wearout due to a passage of time. Their dependent variable is consumer ratings of advertising recall.

2 Like Naik et al. (1998), Bruce et al. (2012) allow for declining effectiveness of advertising through both a concave goodwill production function as well as through the advertising effectiveness multiplier. We do not follow this approach since we are concerned about the high potential for over-fitting the data and relying exclusively on functional form in identifying the wear-out coefficients, especially since we include interaction terms between different stocks of goodwill in explaining consumer purchase behavior.
similar model in which the wearout effects of different advertising “themes” are studied. Their application they study is advertising’s effect on the demand for residential phone service, and the advertising themes include price offer, product offer, reconnection, and reassurance advertisements; the function for effectiveness includes interactions between the amount of advertising for each theme with the aggregate amount of advertising by the other themes, and the authors find negative interaction effects.

Bruce et al. (2012) is one of the only papers we are aware of that incorporates interactions between paid or owned media and earned media in driving consumer purchases, in their case in the form of user reviews; the extant multi-channel literature focuses mostly on interactions effects of paid media or between paid media and promotions. In particular, there is a large gap in the literature with regards to interaction between paid and owned media with social media exposures. However, recent evidence has demonstrated the value of online discussions in driving sales, including everything from television ratings (Godes and Mayzlin 2004) and book sales (Chevalier and Mayzlin 2006) to website traffic (Godes and Mayzlin 2009) and social media participation (Trusov et al. 2009) to hotel consideration (Vermeulen and Seegers 2009) and videogame sales (Zhu and Zhang 2010). These findings are all in the domain of experience goods, where transfer of information between consumers via online word-of-mouth could drive the effectiveness of earned media. It is less clear whether earned media will be effective in driving sales of other types of products.

Two papers which do explore interactions with social media include Onishi and Man-
Onishi and Manchanda (2011) study the effect of the volume of blog activity and television advertising on movie sales in Japan, jointly modeling the demand for movies and blog activity using a reduced form approach. Stephen and Galak (2012) study the interaction between traditional and social earned media (news mentions and blog activity) in driving participation in micro-financing, using a multivariate autoregressive double Poisson model. Neither paper explicitly models effectiveness of media in augmenting goodwill.

It is important to note that in models which include the effect of past advertising on demand through goodwill, there are actually two ways in which advertising interactions may be realized. The first is through the production of advertising goodwill as in Bruce et al. (2012) and Bass et al. (2010). The intuition here is that advertising or social media exposures from different channels may increase or decrease each others’ effectiveness, through attention, recall ability, etc. For example, TV advertising could increase the impact of Facebook exposures on Facebook exposure goodwill by increasing the salience of the brand. The other way the different types of media may interact is in the demand function. Onishi and Manchanda (2011) model the interaction between two different stock variables in explaining demand, and find a positive interaction effect between cumulative advertising and cumulative blog activity. This is in contrast to the DLM models of Bruce et al. (2012) and Bass et al. (2010) who model interactions between extemporary marketing in augmenting goodwill but use a single goodwill stock in explaining demand.

There is no reason why a single interaction needs to be selected a priori - the data can
separately identify interaction effects for both contemporaneous and cumulative advertising. However, we are aware of no papers which allow for both interactions between different types of media in augmenting goodwill as well as in driving consumer demand. Allowing for both types of interactions are important, since the different assumptions regarding where the interactions occur have implications regarding the pulsing behavior of firms. A positive interaction effect in the augmentation of goodwill would increase the benefits of simultaneous pulsing, whereas the opposite could be the case if the positive complementarity occurs between different stocks of the different marketing mix variables.

3 Model

We model the effects of both social media and traditional advertising exposures on purchase behavior (using a multivariate Logit model) through their respective goodwill stocks, which decay over time. We depart from the literature by allowing goodwill to be specific to different forms of advertising. Therefore, a firm is not only able to accumulate goodwill through many different channels, including social media or traditional forms of advertising such as TV ads, but the effect of the goodwill stocks on sales also vary by channel. Our inclusion of a separate goodwill stock from social media allows for differential effects in both the immediate and long term response to social media exposures. Furthermore, by

\[^4\text{Pulsing may be optimal in monopoly due to the S-shaped response curve (Simon 1982, Mahajan and Muller 1986, Feinberg 1992) or in the case of oligopolistic competition (Villas-Boas 1993, Dube et al. 2005) for certain demand functions.}\]
using different goodwill stocks, we allow the decay rates (forgetting) to vary by channel, which is important if long-run advertising effectiveness differs considerably among different forms of marketing, as shown by Vakratas and Ma (2005).

We allow for two levels of interactions across media platforms. The first set of interactions is in the creation of platform-specific advertising goodwill stocks which is augmented by that platform’s contemporaneous advertising and its interactions with the other platform goodwill stocks. The second level of interactions occur between all of the goodwill stocks in the consumer utility function. We also allow ad exposures at the individual level to be a function of the brand-specific demand shock and then individual’s utility function parameters. A schematic of our full model is shown in Figure 8.

3.1 Demand Model

We model the effect of platform-specific exposures through their respective goodwill stocks. Advertising goodwill is a stock built from past advertising and is subject to depreciation over time; it can be thought of as a consumer-specific brand equity. To maintain a constant level of goodwill, advertising is needed in each period. This method of incorporating both short and long term effects of traditional advertising is common in the structural advertising literature. We depart from the literature by allowing goodwill to be specific to different forms of advertising. Therefore, a firm is not only able to accumulate goodwill through many different channels, including social media or traditional forms of
advertising such as TV ads, but the effects of the goodwill on sales also vary by channel. Our inclusion of an additional goodwill stock from social media allows for differential effects in both the immediate and long term response to social media exposures, allowing us to test the claim that the primary benefit of social media is through increased brand equity.

A firm can use advertising of form $f$ to create the augmented goodwill stock of form $f$ for period $t$:

$$g_{ijft}^a = g_{ijft} + \psi(A_{jft}, A_{jft} \star g_{ij,-ft}),$$

(2)

where $A_{jft}$ is advertising for product (or sub-brand) $j$ using form $f$. Following Dube et al. (2005), $\psi$ is the goodwill production function for advertising of form $f$; $g_{ij,-ft}$ is the vector of augmented goodwill of forms other than form $f$. The production of goodwill of form $f$ depends on the amount of current media exposures of this form and its interaction with the amount of other forms of exposures. We use:

$$\psi(A_{jft}, g_{ij,-ft}) = 1 - \exp\left(-A_{ijft} - \sum_{f' \neq f} \gamma_{ijf'} A_{ijft} g_{ijf't}\right),$$

(3)

to allow for declining effectiveness of current period advertising. We allow the effectiveness of current period advertising in creating goodwill of form $f$ to depend on the goodwill stocks of the other advertising forms. The $\gamma_{ijf'}$ allow the current levels of goodwill for the other platforms to affect the goodwill production for platform $f$.

This form is more attractive than some of the alternatives, such as translog, because
our specification nests the single goodwill stock model. We assume that the curvature of the goodwill production function is the same across platforms. In contrast, the translog formulation assumes declining advertising effectiveness which is form specific. If we include interaction terms inside of the log, then they can be separated since $\log(AB) = \log(A) + \log(B)$ and so we are not really modeling interactions. If we include the interaction terms outside of the logs, such as $\log(A) \log(B)$, then the model no longer nests the single goodwill stock model. We will compare our decay rates to assess whether or not a model with a single goodwill stock would be sufficient in modeling demand, which is standard (although the researcher would still need to know how much to weight exposures across platforms).

Following Dube et al. (2005) (while allowing form-specific decay rates), let a firm’s augmented goodwill of form $f$ depreciate and become the goodwill stock in the next period, $t + 1$:

$$g_{ijf,t+1} = \lambda_f g_{ijf,t}. \quad (4)$$

We assume that $0 < \lambda_f < 1$, so that goodwill depreciates in expectation. We do not include a stochastic term for computational reasons - because we are estimating the parameters of the goodwill production function, and the decay parameters, we need to recalculate the goodwill stocks every iteration of the estimation procedure. Adding stochastic depreciation makes this too computationally burdensome.

We use an individual-level consumer utility model which will depend on their current stock of advertising goodwill for each form of advertising. Let consumer $i$’s utility for
product $j$ at time $t$ be given as:

$$u_{ijt} = X'_{ijt}\beta_i + \alpha^p_ip_{jt} + \alpha^A_i\left(1 - \exp\left(-\sum_{f=1}^F \beta_ifg_{ijft} - \sum_{f'>f} \beta_if'g_{ijft}g_{ijft'}\right)\right) + \xi_{ij} + \eta_{jt} + \epsilon_{ijt},$$

(5)

where $X_{ijt}$ are consumer characteristics (past purchasing behavior) and product characteristics (product type, brand, etc.), $p_{jt}$ is price, $g_{ijft}$ is consumer-specific goodwill for product $j$ due to form $f$, $\xi_{ij}$ are consumer-product specific intercepts, and $\eta_{jt}$ captures time varying demand shocks for product $j$. $\alpha^p_i$ is the individual-level price sensitivity and $\alpha^A_i$ is the advertising goodwill sensitivity. The $\beta_{if}$ determine the relative effectiveness of advertising for different platforms, and the $\beta_{if'f'}$ are the interaction terms. Our functional form mirrors that used for the goodwill production function in order to limit arbitrary differences in functional form in driving identification.

We use a multivariate Logit model of demand, i.e. we assume that $\epsilon_{ijt}$ is distributed type 1 extreme value. A consumer chooses to buy project $j$ if his or her utility for the product exceeds the utility of the outside alternative. We use this model because it is clear from the data that consumers are not making a discrete choice between brands - many transactions include multiple brands purchased. Since we model the decision to purchase a brand conditional on purchasing in the category due to data restrictions, the outside option is the choice of the generic or minor brands. The probability that consumer $i$ purchases product $j$ at time $t$ is:

$$Pr_{ijt}(y_{ijt} = 1) = \Phi(u_{ijt}),$$

(6)
where we normalize the error term so that $\Phi$ is the standard Logit cumulative distribution function. Our model of demand is a conditional demand model: we only model brand choice conditional on purchasing in the category. Given the category we study, this is more reasonable that assuming that the consumer makes a single purchase decision every time period, where the time period is arbitrarily set (often weekly or monthly purchase occasions are assumed). This allows us to calculate goodwill using daily data, ensuring that we include all relevant exposure data in estimating the demand parameters; when aggregating data at the weekly or monthly level, it is necessary to either use advertising goodwill that accumulated prior to the current period, or include the current period level, even though the consumer may not have been exposed before the purchase decision occurred. One final benefit of our method is we are able to condition the purchase decision on the set of available brands in the store on that purchase occasion.

3.2 Model of Advertising

One of the greatest challenges is estimating advertising effects is the endogeneity of firms’ advertising decisions. For example, if firms decide to advertise during periods of peak demand due to seasonality or other expected demand shocks, then the estimated coefficient will be biased upwards, leading econometricians to overestimate the benefits of the advertising. We can overcome this issue somewhat using variation in customer-level advertising exposures; however, since the amount of exposures does still depend on the firm’s level of advertising (in addition to consumer-level heterogeneity), we need to ex-
plicitly model advertising exposures at the consumer level. Furthermore, advertising can be targeted to consumers of different types, especially on the digital media formats, another reason for modeling advertising exposures.

We assume that advertising exposures follow a Poisson distribution with parameter $\phi_{ijft} > 0$ which describes the rate at which consumer $i$ is exposed to advertising by brand $j$ of form $f$. Recall $\zeta_{ij}$ (contained in $\theta_i$) is an individual-specific coefficient that captures the amount of time spent on platform $f$ by consumer $i$. The rate of exposure takes the form:

$$\phi_{ijft} = f(\theta_i) A_{jft},$$

(7)

where $A_{jft}$ is the amount of advertising done by brand $j$ of form $f$, and $f(.)$ is a nondecreasing function in $w_{ift}$ and targeting may be possible as a function of consumer’s advertising response parameters, $\theta_i$, similar to the specification in Manchanda et al. (2004). We assume:

$$f(\theta_i) = \exp(\zeta_{ij}^0 + [\alpha_{f}^P, \alpha_{f}^A, \beta_{ij}] * \zeta_{ij}^\theta + \delta_{ij} + \epsilon_{w_{ift}}),$$

(8)

where the $\zeta_{ij}^\theta$ vector describes the targeting. We assume advertising by brand $j$ (of category one) is a fraction of category advertising:

$$A_{jft} = A_{1ft} \exp(\zeta_{ij} + \zeta_{ij}^\eta + \epsilon_{jft}),$$

(9)

where $A_{1ft}$ is category one advertising, with the “1” superscript designating that this is category 1, the focal category for the analysis. The brand dummies, $\zeta_{ij}$, explain the baseline
amount of advertising by brand \( j \) on platform \( f; \eta_{jt} \) is the unobserved (to the econometrician) aggregate demand shock for brand \( j \) which is the source of the endogeneity, and \( \varepsilon_{jft} \) is an exogenous supply shock. We also assume that category advertising for category \( c \) is function of the cost of advertising on the platform, \( c_{ft} \):

\[
A_{f t}^c = \left( \exp \left( h(c_{ft}) + \varepsilon_{jft}^c \right) \right)^{\gamma^c},
\]

where \( h(.) \) is any non-increasing function in the marginal costs of advertising \( c_{ft} \) and \( \varepsilon_{jft}^c \) is an exogenous supply shock. We use total exposures by category for all households as our measure of category advertising.

Now we have no information concerning advertising costs. However, we can write category one advertising as a function of category two advertising since the advertising costs are independent of category:

\[
A_{1 ft}^1 = (A_{2 ft}^2)^{\zeta_A^2} \exp(\varepsilon_{1 ft}^1 - \varepsilon_{2 ft}^2),
\]

where we define \( \zeta_A^2 = \zeta^{c=2}/\zeta^{c=1} \).

By substitution of (11) and (9) into (7), and defining \( \varepsilon_{ijft} = \varepsilon_{ift} + \varepsilon_{jft} + \varepsilon_{1 ft}^1 - \varepsilon_{2 ft}^2 \), we can now write (7) as:

\[
\phi_{ijft} = \exp \left( \zeta_0^0 + [\alpha_i^p, \alpha_i^A, \beta_{ijf}] \ast \zeta_f^g + \zeta^{A^2} \log(A_{ft}^2) + \zeta_{jft}^n + \zeta_{ft} + \zeta_{ijft}^n + \delta_{if} + \sigma\varepsilon_{ijft} \right). \tag{12}
\]

This equation is used in conjunction with the demand model to control for the endogenei-
ity of advertising. In practice, we find little explanatory value in the epsilon term, so we drop it and allow the stochastic nature of exposures to be purely driven by the Poisson process.

Yang et al. (2003) similarly estimate both a supply and demand side model, but use a more structural interpretation of the supply side coefficients than we will in this paper. Since our only goal in incorporating an exposure model is to control for the endogeneity issues, we use a limited information approach as in Cohen (2013). Rather than assuming correlated demand and supply shocks, we instead allow brand-level advertising to be a direct function of these shocks, as well as a function of the individual-level demand parameters.

3.3 Identification

Our model allows for interactions between different forms of advertising in two ways: First, there may be complementarities between different advertising in the creation of goodwill/brand equity. This is accounted for by allowing interactions in the goodwill production functions. In addition, the effect of goodwill stock for different forms may interact in the consumers’ utility function; furthermore, there is no way to know in advance if this interaction is negative (advertising forms are substitutes) or positive (complements).

Because our model allows for interactions in two ways, we need to be precise regarding the identification of the model. The identification comes from variation in the current
levels advertising by form with variation in the relative goodwill stocks. To take an extreme example, if a firm uses TV advertising in one period and none in the next, and also uses Facebook impressions, there is an interaction in the second period between the goodwill stocks, which affects sales, but no interaction in the creation of goodwill. In the first period when the advertising occurs, there is an interaction in both. So long as advertising expenditures do not correlate perfectly with goodwill, we can estimate separate interaction effects. Time-series variation will identify the decay rates for the different types of advertising.

4 Estimation

4.1 Likelihood

For ease of exposition, we define \( \theta_i \equiv \{ \alpha_i, \beta_i, \{ \beta_{ij}|\forall f \}, \{ \beta_{ijf}|\forall f, f' \}, \{ \xi_{ij}|\forall j \}, \{ \delta_{ij}|\forall f \} \} \) for consumer \( i \), where \( \zeta_{ij} \) is an individual-specific ad exposure intercept for platform \( f \) that will enter the advertising exposure models; this notation will be used throughout the rest of the paper. Define \( \zeta = \{ \zeta_0^f, \zeta_f^0, \zeta_{ff}, \zeta^n \} \). In the following, when we drop a parameter or variable subscript, we are referring to the vector of parameters or variables for all values of the subscript.

Conditional on the parameters, we can write the likelihood of the data for individual \( i \) as:

\[
L_i(X_i, y_i, A_i|\theta_i, \zeta, \gamma, \lambda, \eta) = \prod_i \prod_j \Phi(u_{ij}) \gamma_{ij} \prod_f \phi_{ijf} \exp(-\phi_{ijf})
\]

(13)
where $Y_{ijt}$ are dummy variables indicating brand purchases and $A_{ijf}$ are brand exposures on platform $f$. We can use this expression to write the likelihood of each parameter vector, conditional on the data. Details can be found in the Appendix. We assume that the individual-level parameters are drawn from a normal distribution: $\theta_i \sim N(\delta, \Sigma)$.

We use Markov-Chain Monte Carlo (MCMC) methods to estimate the model. The estimation procedure is as follows:

1. Start with initial values for $\lambda, \gamma, \theta, \delta, \Sigma, \eta, \zeta$
2. For each $i$: Draw $u_{ijt}|\{X_{ijft}, Y_{ijt}|\forall t\}, \theta_i, \lambda_i, \eta$
3. For each $i$: Draw $\theta_i, \delta_i, \lambda_i|\{u_{ijt}, A_{ijft}|\forall j, f, t\}, \zeta, \eta$
4. For each $j, f$: Draw $\zeta_{jf}|\{A_{ijft}|\forall i, t\}, \delta_i, \eta_j$
5. For each $j, t$: Draw $\eta_{jt}|\{u_{ijt}, A_{ijft}|\forall i, f\}, \zeta$
6. Draw $\delta|\theta, \Sigma$
7. Draw $\Sigma|\delta, \theta$

4.2 Simulation results

We demonstrate our ability to recover model primitives by providing simulation results. We find that using starting values for the unobserved demand shocks of the correct sign are very important in being able to estimate their true values. We find that using maximum likelihood estimation (assuming homogenous consumers) to establish the starting values is all that is required in practice.
We simulated data with 300 households and 50 time periods using a data generating process in which the level of exposures were a function of the household’s utility function coefficients, an independent regressor (in our model, the advertising in the other category), and the unobserved demand shock. We include three variables (price and advertising on two platforms) plus an intercept in the consumer utility function, and we have three product alternatives. We estimate the model first without controls for endogeneity and then using our method. The household utility parameters were drawn from a normal distribution with mean $[1; 2; -2; 0]$ with two demographic variables explaining some of the heterogeneity. The exposure model assumed that households are exposed according to a Poisson process where the hazard rate is: 

$$\lambda_i = 0.5 \ast \eta \ast [1; \beta_i; \eta_{jt}; Z_{ft}]$$

where $\eta$ is a vector of ones and we use the same notation for the indices as before. The results are in Table 2.

The true and estimated distributions of $\beta_i$ are in Figure 8. Eta was drawn from a normal distribution with mean zero and variance of one. Of the 150 $\eta_{jt}s$, only seven were of the incorrect sign with a mean magnitude of 0.0695 and a maximum of 0.1473. The true and estimated distributions of $\eta_{jt}$ are in Figure 8 and we plot each of the shocks, both true and estimated, in Figure 8. It is clear we can recover the shocks using our methodology.

Using our data, a single iteration of the MCMC chain takes over a minute. The estimation procedure is computationally intensive because we need to recalculate the goodwill stocks every iteration of the chain, we need to also loop through individuals in drawing the individual-level parameters, and we need to loop through brand-time combinations.
in drawing the unobserved demand shocks.

5 Data

The data set, provided by a Fast Moving Consumer Goods (FMCG) company, consists of four overlapping panels that were tracked through 2010 and 2011. The respondents were all recruited within a Western European country and the data contains their advertising exposure and purchases for two unrelated product categories.

Transaction Data The transaction data include purchases by approximately 30,000 households in the focal category we use for the analysis for 2010 and 2011 in the 26 retailer stores in the country. The original purchase data records the date and the store of a given purchase trip, the integer identifier for the product that was purchased, the total price that a household paid for the total purchases of a single product in a given day, whether the promotion was applied at the time of purchase, which umbrella brand and brand the product belongs to, units of a particular product that the panelist purchased that day and the size of the packaging for the product purchased. Figure 5 shows the histogram of average number of days between two shopping trips in the year 2011 for the whole sample. The majority of households (around 68%) on average make a shopping trip for the category 1 product between a week to a month. Figure 6 shows the histogram of percentage of shopping trips with multiple brand purchases in the year 2011 for the whole sample. Among 22,692 households in the whole sample, 18,959 (around 84%) of them has some
shopping trips with multiple brand purchases, and 9,250 (around 41%) of them has more than 20% of their shopping trips being trips with multiple brand purchases. This is the reason for using a multivariate Logit demand model rather than a discrete choice model - we model the decision to purchase the available brands in the store, conditional on a purchase being made in the category at the store.

**Facebook Exposure Data**  In the original dataset, one observation tracks a message a user sees in his/her newsfeed, ticker; and others’ timelines/walls. These are the messages that are actually displayed on his/her computer, not all the messages Facebook makes available to him/her. The original Facebook dataset tracks the top 32 most “Liked” brands in the country where the panel took place. 4 out of these 32 umbrella brands belong to category 1.

7,538 households in the whole sample have information of their umbrella brand exposure on Facebook. 2,823 out of this 7,538 households (around 37%) have exposure to the 4 umbrella brands belonging to category 1. Figure 7 shows the histogram of number of umbrella brands households has been exposed to on Facebook. More than half of the households has been exposed to less than 5 umbrella brands among the top 32 most “liked” brands in the country.

For each household, if there is an exposure on one of the 32 umbrella brands, we observe which umbrella brand it is, the date of exposure to brand message, whether the contact type is owned or earned (contacts are “owned” if the message displayed on the panelist’s newsfeed or a wall was send/written by the brand page; “earned” contacts
are contacts to brand names in a message written by a Facebook user: the panelist or a friend.), and the notification type of the message (For owned contacts, notification types include brand like on profile, brand like on newsfeed, brand like on timeline, brand like on ticker, brand message, liked message and shared message; for earned contacts, notification types includes status and comment).

Table 1 tabulates types of messages households are exposed to on Facebook for both all the top 32 umbrella brands and the 4 umbrella brands belonging to category 1. In both cases, the majority of messages come from friends’ status and comments. Figure 9 shows the percentage of friends’ messages belonging to each umbrella brand in category one, by type. Figure 10 presents the aggregate weekly brands exposure on Facebook for the four umbrella brands in category 1 that have Facebook brand exposure information. The aggregate weekly brands exposure is the sum of total number of exposure for a given brand across all notification types and households.

To see the supply side relationship between the number of exposures for our two categories, we plot total category exposures on Facebook over time for our focal category brands and the other top 30 brands on Facebook in Figures 12 and 12.

One issue with the data is that we only observe the top 30 liked brands’s Facebook brand exposure, 4 of which belong to category 1. So we do not have Facebook brand exposure information of other brands belonging to category 1. This lack of data will not cause significant bias in the result if other brands have relatively small Facebook brand exposure compared with the brands with big Facebook exposures and other brands’ Face-
book brand exposure are not highly correlated with brands with big Facebook exposures.

**Online Exposure Data** In the original dataset, one observation is an instance where a banner ad (both static and dynamic), pre-roll video (video commercial preceding a YouTube, or similar service, video) was presented to a panelist. These include both the company’s and its competitor’s brands in 2011 and only the company’s brands in 2010.

In total, 8,733 households have exposure to the 5 umbrella brands belonging to the sponsor in 2010; 10,468 households have exposure to the 5 umbrella brands belonging to the company and another 5 brands belonging to its competitors in 2011. Figure 13 shows an exposure histogram for the 8,733 households in the online dataset in 2010 and the 10,468 households in 2011. For each household, if there is an exposure online, we observe which umbrella brand it is, the date of exposure to brand message, whether the ad type is Banner, Pre-Roll or Facebook Banner. Table 6 tabulates types of ads online for the year 2010 and the year 2011.

Figure 14 presents the aggregate weekly brands exposure online for the five umbrella brands in category 1 that belong to the company. The aggregate weekly brands exposure is the sum of total number of exposure for a given brand across all ad types and households. Figure 15 presents the aggregate weekly brand exposure online for one of the major brands belonging to the company (umbrella brand 32) and five other brands belonging to its competitors in 2011 - the huge disparity in the level of online advertising across firms makes it crucial that we allow for brand-specific intercepts in our exposure model.
We plot category level online exposures in Figure 16.

**TV Exposure Data** Each time an ad is displayed on a household’s TV, we observe time and date the ad was delivered, which ad copy the ad belongs to, the percentage of the ad which the person saw (e.g., viewing 10 seconds of a 30 seconds ad would be recorded as 0.33), which umbrella brand the ad belongs to and which brand the ad belongs to. In total, 4,831 households are exposed to 41 brands (35 umbrella brands) in 2010, and 5,747 households are exposed to 42 brands (34 umbrella brands) in 2011. Figure 17 provides a histogram for the number of exposures in 2010 and 2011 for the households that are also in the online dataset. In both years, more than half of the households are exposed to at least 30 different umbrella brands.

We plot category level TV exposures in Figure 18.

**The Estimation Sample** Of the 19,062 households with demographics and purchase information in the year 2011, we have information regarding their online and TV exposures for 8,293, and Facebook exposures for 3,347. Of these 3,337 households, all but ten make at least one purchase in the focal category. Our demographic information includes number of children under 18, type of work and employment status of head of household, age of head of household, home ownership, size of household, income of household, education of head of household and the type of home of the household. In Appendix A, we compare the number of trips made by households, the time between trips, and the number of brands in the focal category bought per trip (conditional on at least one being
purchased) for the estimation sample and the full sample, using kernel densities. The estimation sample is in no way systematically different than the full sample, as expected since the individuals in the data set are randomly selected.

6 Results and Discussion

We first estimate the model without the inclusion of the advertising exposure model. This will allow us to assess the importance in controlling for the endogeneity of advertising. The mean and standard deviation across individuals for the individual-level parameters are in the first two columns of Table 8. We find the expected negative effect of price and positive effect of TV goodwill on consumer utility. It is hard to assess the impact of online and banner ad exposures on utility since they gave negative direct effects but positive effects in their interactions - for the results of the full model we calculate total advertising elasticities accounting for these interactions.

When we incorporate the exposure model to control for the endogeneity of advertising due to correlation in expected demand and the resulting advertising by the firms, we find that the estimated price elasticity increases in magnitude by 17%, although the advertising coefficients are largely unchanged. However, when we extend the model further to allow for ad exposures to be a function of consumers’ utility parameters due to targeting, we find that the price estimate is similar to that when ignoring all sources of endogeneity, but that the advertising coefficients are altered. The coefficient on total advertising decreases when controlling for the fact that ads may be targeted. There is an increase in the
coefficient on television advertising to offset part of this difference - the marginal effect of a TV ad with no advertising is the product of the two coefficients, which changes from 0.61 to 0.53 for television. The same is true for online ads but not for Facebook exposures. In comparing the estimates for all parameters when only using the demand model versus allowing for both sources of endogeneity (both due to aggregate levels of advertising and targeting), we find that it is important to control for these factors.

The distributions of the utility parameters for the full model can be found in Figure 8. Most of the price parameters are less than zero as expected, and the TV are mostly above zero. To better see the effect of the different types of exposures and to get a sense of what the magnitude of the effects are, we plot histograms of individual-level elasticities (including the interaction effects), conditional on exposures on that platform in Figure 8. The average price elasticity is -0.150, with considerable variation across consumers. This may seem like a low amount, but one thing to remember is that we are using a conditional demand model, where we condition on a category purchase occurring. Category expansion effects are therefore not included. With 52 different brand is a horizontally differentiated industry with low price items, it is not too surprising that demand may be relatively inelastic. We find an average TV elasticity of advertising of 0.014, conditioning on household with at least some exposures. There is a negligible average elasticity of online exposures and a negative one for Facebook, -0.0248. One thing not controlled for is the valence of Facebook exposures - since these can be positive or negative, there is no reason why would would only expect positive effects of exposures.
Although we find a negligible average effect of online banner ad goodwill on consumer utility, this does not mean there is no effect at all. In our model, we also incorporate interaction effects in the creation of goodwill. The mean and standard deviations of the goodwill production function and the decay parameters can be found in Table 2 and the distributions are shown in Figure 8 and Figure 8. The mean interaction effect of online banner ads on the creation of TV goodwill is 0.587, which means that at a goodwill level of just two exposures, TV ads are twice as effective in building goodwill; likewise, TV goodwill has positive effects on the creation of online goodwill. Facebook exposures have a small average effect in the creation of TV and online goodwill stocks, and the considerable heterogeneity in the effect make it difficult to say anything definitive about their effect. Decay rates are lowest (i.e. carryover is highest) for online exposures and highest for Facebook.

The coefficient estimates for the exposure model can be found in Table 8 and Figure 8. The positive coefficient on the price coefficient indicates that less price sensitive consumers are more likely to see TV ads, although the coefficient is not significant. In contrast, price sensitive consumers are much more likely to have Facebook exposures. Consumers who are more affected by any type of exposures are more likely to have Facebook exposures, but there is a counteracting negative coefficient on the effect of Facebook exposures on their likelihood of being exposed. Not surprisingly, consumers see many more Facebook exposures in periods of peak demand, although there is no significant effect of the demand shocks on TV and online exposures. There is a positive effect of ads in
the other category(ies) on exposures for TV and Facebook, indicating that there are shocks on the supply side (cost shocks for TV, probably platform shocks for Facebook) that affect exposures irregardless of the category. In contrast, the effect is negative for online banner ads, but this is consistent if the main determinant of online costs are the demand for advertising.

7 Concluding Remarks

In this paper, we estimate the effectiveness of platform-specific goodwill in driving consumer demand in a frequently purchased CPG category using single-source data. We allow for interaction effects in the creation of goodwill and in the consumer utility function and find a positive interaction effect in the creation of TV and banner ad goodwill stocks. We shed light on the recent debate over the effectiveness of social media exposure by addressing the following: i) How social media, specifically positive impressions on Facebook, affect brand purchases ii) How different consumers respond differently to social media exposure and traditional forms of advertising iii) Whether there are differences in the “quality” of goodwill created through the different channels, which results in different decay rates for the different types of goodwill, and iv) How goodwill from social media exposures interacts with other advertising goodwill.

We find that TV advertising is by far the most effective medium in affecting consumer demand, consistent with findings by [Lovett and Staelin (2012)]. We find that heterogeneity in decay rates for TV and Facebook goodwill lead to the highest average carryover
effect for banner ad exposures. The differences in the decay rates lead us to conclude that a single goodwill stock model would not sufficiently capture reality, and by allowing for different goodwill stocks we can also estimate their asymmetric effects on demand. Although we do not find a positive impact (on average) of online goodwill on utility, we do find that online exposures can increase the effectiveness of TV advertising. Because we find a positive interaction effect in the goodwill production functions, this would imply that advertisers should run TV ads at the same time or just after online campaigns to maximize goodwill creation.

Our findings on the effectiveness of Facebook exposures are less positive. Last year Facebook launched a “Sponsored Stories” feature that lets advertisers rebroadcast users’ positive posts on the site’s main news feed to highlight them. Advertisers pay Facebook $8 every time an ad called “Sponsored Story” is viewed 1,000 times in the main news feed. So $1 million would buy 125 million views, or impressions. The same amount of money could buy two 30-second ads on “American Idol”, or two days on Yahoo’s home page for a large ad with rich media. The argument has been made by some that the value of a social media presence is not merely the immediate sales response but also the increase in brand equity which results from allowing consumers to connect on a personal level with a brand. Facebook serves as a platform for engaged consumers to converse about the brands they “like” or dislike, and how they interact with them. For the average consumer, we do not find any evidence of a positive impact of Facebook exposures, although there is considerable heterogeneity.
The obvious limitation of our paper from a substantive perspective is the lack of experimental data, which would ensure exogenous aggregate advertising and no targeting (although individual level exposures would still depend on consumer characteristics which predict their time spent on the platform). However, our methodology in estimating the unobserved demand shocks and allowing firms to set aggregate advertising as a function of these shocks, in conjunction with the Manchanda et al. (2004) method of allowing targeting as a function of demand parameters, allow us to control for multiple sources of endogeneity, specifically changes in category advertising as a function of expected demand and individual level targeting. While not perfect in controlling for some sources of endogeneity (such as brand-specific targeting functions), our method provides a large improvement over two options often taken, namely ignoring endogeneity entirely or abandoning interesting questions altogether. The method relies on variation in the level of category exposures that can be related to an exogenous shifter, in our case the exposures in an unrelated category which will reflect unobserved costs of advertising, and in the single-source nature of the data which allow us to control for the individual-level targeting.
8 Tables and Figures

Table 1: Tabulation of type of messages on Facebook

<table>
<thead>
<tr>
<th>NOTIFICATION_TYPE</th>
<th>brandpage</th>
<th>friend</th>
<th>self</th>
<th>Total</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>brand like on newsfeed</td>
<td>0</td>
<td>11,346</td>
<td>0</td>
<td>11,346</td>
<td>1.25%</td>
</tr>
<tr>
<td>brand like on profile</td>
<td>0</td>
<td>6,726</td>
<td>4,638</td>
<td>11,364</td>
<td>1.25%</td>
</tr>
<tr>
<td>brand like on ticker</td>
<td>0</td>
<td>189</td>
<td>0</td>
<td>189</td>
<td>0.02%</td>
</tr>
<tr>
<td>brand like on timelin</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>7</td>
<td>0.00%</td>
</tr>
<tr>
<td>brand message</td>
<td>171,494</td>
<td>0</td>
<td>0</td>
<td>171,494</td>
<td>18.93%</td>
</tr>
<tr>
<td>comment</td>
<td>0</td>
<td>318,070</td>
<td>6,383</td>
<td>324,453</td>
<td>35.81%</td>
</tr>
<tr>
<td>liked message</td>
<td>0</td>
<td>1,765</td>
<td>15</td>
<td>1,780</td>
<td>0.20%</td>
</tr>
<tr>
<td>liked message fullstory</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0.00%</td>
</tr>
<tr>
<td>shared message</td>
<td>0</td>
<td>717</td>
<td>32</td>
<td>749</td>
<td>0.08%</td>
</tr>
<tr>
<td>status</td>
<td>0</td>
<td>367,051</td>
<td>17,710</td>
<td>384,761</td>
<td>42.46%</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

171,494 705,872 28,778 906,144 100%

Percentage

18.93% 77.90% 3.18% 100%

<table>
<thead>
<tr>
<th>NOTIFICATION_TYPE</th>
<th>brandpage</th>
<th>friend</th>
<th>self</th>
<th>Total</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>brand like on newsfeed</td>
<td>0</td>
<td>1,188</td>
<td>0</td>
<td>1,188</td>
<td>4.03%</td>
</tr>
<tr>
<td>brand like on profile</td>
<td>0</td>
<td>591</td>
<td>356</td>
<td>947</td>
<td>3.21%</td>
</tr>
<tr>
<td>brand like on ticker</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>9</td>
<td>0.03%</td>
</tr>
<tr>
<td>brand like on timelin</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.00%</td>
</tr>
<tr>
<td>brand message</td>
<td>4,868</td>
<td>0</td>
<td>0</td>
<td>4,868</td>
<td>16.52%</td>
</tr>
<tr>
<td>comment</td>
<td>0</td>
<td>11,005</td>
<td>180</td>
<td>11,185</td>
<td>37.96%</td>
</tr>
<tr>
<td>liked message</td>
<td>0</td>
<td>30</td>
<td>0</td>
<td>30</td>
<td>0.10%</td>
</tr>
<tr>
<td>liked message fullstory</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.00%</td>
</tr>
<tr>
<td>shared message</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.00%</td>
</tr>
<tr>
<td>status</td>
<td>0</td>
<td>10,884</td>
<td>354</td>
<td>11,238</td>
<td>38.14%</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4,868 23,707 890 29,465

Percentage

16.52% 80.46% 3.02% 100%
Table 2: Simulation Results

<table>
<thead>
<tr>
<th>True value</th>
<th>demand model only</th>
<th>with exposure model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0166</td>
<td>0.4745</td>
<td>1.0499</td>
</tr>
<tr>
<td>1.9641</td>
<td>1.2681</td>
<td>1.9920</td>
</tr>
<tr>
<td>-1.9826</td>
<td>-1.4073</td>
<td>-2.1635</td>
</tr>
<tr>
<td>0.0316</td>
<td>0.5111</td>
<td>0.0597</td>
</tr>
</tbody>
</table>

Table 3: Estimation Results, Utility Parameters

<p>| Model: demand only without targeting full model |</p>
<table>
<thead>
<tr>
<th>Variable</th>
<th>mean std. dev.</th>
<th>mean std. dev.</th>
<th>mean std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>price</td>
<td>-0.760 0.832</td>
<td>-0.893 0.920</td>
<td>-0.787 0.714</td>
</tr>
<tr>
<td>all ad.</td>
<td>0.640 0.912</td>
<td>0.674 0.983</td>
<td>0.504 0.447</td>
</tr>
<tr>
<td>tv gw</td>
<td>0.873 1.239</td>
<td>0.909 1.347</td>
<td>1.054 0.902</td>
</tr>
<tr>
<td>online gw</td>
<td>-0.208 1.246</td>
<td>-0.254 1.369</td>
<td>-0.104 0.883</td>
</tr>
<tr>
<td>FB gw</td>
<td>-0.751 1.354</td>
<td>-0.793 1.458</td>
<td>-0.775 1.143</td>
</tr>
<tr>
<td>tv gw x online gw</td>
<td>1.1984 1.357</td>
<td>1.254 1.488</td>
<td>1.123 1.076</td>
</tr>
<tr>
<td>tv gw x FB gw</td>
<td>0.4206 1.366</td>
<td>0.432 1.483</td>
<td>0.456 0.925</td>
</tr>
<tr>
<td>online gw x FB gw</td>
<td>0.0832 1.573</td>
<td>0.092 1.684</td>
<td>0.033 1.273</td>
</tr>
</tbody>
</table>

Table 4: Full Model Estimation Results, Goodwill Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>mean std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>carryover</td>
<td>0.664 0.209</td>
</tr>
<tr>
<td>TV gw</td>
<td></td>
</tr>
<tr>
<td>x O gw</td>
<td>0.587 0.840</td>
</tr>
<tr>
<td>x FB gw</td>
<td>-0.222 1.167</td>
</tr>
<tr>
<td>Online gw</td>
<td></td>
</tr>
<tr>
<td>carryover</td>
<td>0.956 0.058</td>
</tr>
<tr>
<td>x TV gw</td>
<td>0.114 1.103</td>
</tr>
<tr>
<td>x FB gw</td>
<td>-0.189 1.110</td>
</tr>
<tr>
<td>FB gw</td>
<td></td>
</tr>
<tr>
<td>carryover</td>
<td>0.597 0.287</td>
</tr>
<tr>
<td>x TV gw</td>
<td>0.202 1.143</td>
</tr>
<tr>
<td>x O gw</td>
<td>-0.311 1.042</td>
</tr>
</tbody>
</table>
Table 5: Full Model Estimation Results, Exposure Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>price coef.</th>
<th>ad. clef.</th>
<th>platform ad coef.</th>
<th>demand shock</th>
<th>other category ad.</th>
</tr>
</thead>
<tbody>
<tr>
<td>TV</td>
<td>0.0267</td>
<td>0.0176</td>
<td>-0.0148</td>
<td>-0.0200</td>
<td>0.0242</td>
</tr>
<tr>
<td></td>
<td>(0.0254)</td>
<td>(0.0145)</td>
<td>(0.0152)</td>
<td>(0.0307)</td>
<td>(0.0129)</td>
</tr>
<tr>
<td>online</td>
<td>-0.0051</td>
<td>0.0158</td>
<td>0.0019</td>
<td>0.0097</td>
<td>-0.1150</td>
</tr>
<tr>
<td></td>
<td>(0.0192)</td>
<td>(0.0129)</td>
<td>(0.0145)</td>
<td>(0.0301)</td>
<td>(0.0089)</td>
</tr>
<tr>
<td>FB</td>
<td>-0.5559</td>
<td>0.9320</td>
<td>-0.8203</td>
<td>1.4519</td>
<td>0.1580</td>
</tr>
<tr>
<td></td>
<td>(0.0618)</td>
<td>(0.0504)</td>
<td>(0.0678)</td>
<td>(0.0561)</td>
<td>(0.0566)</td>
</tr>
</tbody>
</table>

Values in parenthesis are standard deviation across draws.
Figure 1: Schematic of Model
Table 6: Tabulation of type of ads online

<table>
<thead>
<tr>
<th>Ad type online in 2010 whole sample</th>
<th>umbrella brand id</th>
<th>Banner</th>
<th>Pre-Roll</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>140,341</td>
<td>0</td>
<td>140,341</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>47,458</td>
<td>0</td>
<td>47,458</td>
<td></td>
</tr>
<tr>
<td>54</td>
<td>130,062</td>
<td>0</td>
<td>130,062</td>
<td></td>
</tr>
<tr>
<td>67</td>
<td>50,359</td>
<td>0</td>
<td>50,359</td>
<td></td>
</tr>
<tr>
<td>843</td>
<td>62,211</td>
<td>317</td>
<td>62,528</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>430,431</td>
<td>317</td>
<td>430,748</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ad type online in 2011 whole sample</th>
<th>umbrella brand id</th>
<th>Banner</th>
<th>Facebook</th>
<th>NULL</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>30,075</td>
<td>18,781</td>
<td>0</td>
<td>48,856</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>23,608</td>
<td>0</td>
<td>2,521</td>
<td>26,129</td>
<td></td>
</tr>
<tr>
<td>54</td>
<td>27,576</td>
<td>0</td>
<td>0</td>
<td>27,576</td>
<td></td>
</tr>
<tr>
<td>67</td>
<td>25,533</td>
<td>0</td>
<td>0</td>
<td>25,533</td>
<td></td>
</tr>
<tr>
<td>838</td>
<td>185</td>
<td>0</td>
<td>0</td>
<td>185</td>
<td></td>
</tr>
<tr>
<td>839</td>
<td>1,552</td>
<td>0</td>
<td>0</td>
<td>1,552</td>
<td></td>
</tr>
<tr>
<td>843</td>
<td>28,078</td>
<td>0</td>
<td>0</td>
<td>28,078</td>
<td></td>
</tr>
<tr>
<td>844</td>
<td>4,008</td>
<td>0</td>
<td>0</td>
<td>4,008</td>
<td></td>
</tr>
<tr>
<td>846</td>
<td>397</td>
<td>0</td>
<td>0</td>
<td>397</td>
<td></td>
</tr>
<tr>
<td>866</td>
<td>776</td>
<td>0</td>
<td>0</td>
<td>776</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>141,788</td>
<td>18,781</td>
<td>2,521</td>
<td>163,090</td>
<td></td>
</tr>
</tbody>
</table>

36
(a) True coefficient values

(b) Demand model only

(c) Demand and exposure model

Figure 2: Simulation estimates of utility parameters
(a) True demand shocks

(b) Estimated demand shocks

Figure 3: Simulation estimates of demand shocks

Figure 4: Demand shocks, true and estimated
Figure 5: Histogram of average number of days between two shopping trips in the year 2011 for the whole sample

This histogram only includes the households who have on average less than 100 days between two shopping trips. Among 22,692 households in the whole sample, 21,718 (around 96%) of them have on average less than 100 days between two shopping trips. 15,342 households (around 68% of the whole sample) have the average number of days between two shopping trips in the range of [7 days, 28 days].
Figure 6: Histogram of percentage of shopping trips with multiple brand purchases in the year 2011 for the whole sample

Among 22,692 households in the whole sample, 9,250 (around 41%) of them has more than 20% of their shopping trips being trips with multiple brand purchases.
Among 7,538 households who have information on their umbrella brand exposure on Facebook, around 21% of them has been exposed to 1 umbrella brand and around 52% of them has been exposed to less than 5 umbrella brands among the 30 most liked umbrella brands on Facebook.
This graph shows for each umbrella brand, how many households have been exposed to it on Facebook. The most popular umbrella brand has 6,793 households while the least popular has only 32 households. The four umbrella brands belonging to category 1 product are ranked 7th (umbrella brand 866), 17th (umbrella brand 859), 28th (umbrella brand 32) and 29th (umbrella brand 846) in terms of popularity and have 2,203, 1,373, 440, 347 households respectively.

This figure shows the percentage of messages belonging to each umbrella brand among 11,005 friends comment message about category 1 product, and the percentage of messages belonging to each umbrella brand among 10,884 friends comment message about category 1 product.
Figure 10: Aggregate weekly brand exposure on Facebook

Figure 11: Total Facebook brandpage exposures for category 1 and other top 30 brands
Figure 12: Total Facebook friend messages exposures for category 1 and other top 30 brands
This graph shows among 8,733 households in the online dataset in the year 2010, what are the percentages of households who have been exposed to 1, 2, 3, 4 and 5 umbrella brands; and among 10,468 households in the online dataset in the year 2011, what are the percentage of households who have been exposed to 1, 2, 3, 4, 5, 6, 7, 8 and 9 umbrella brands.
Figure 14: Aggregate weekly brand exposure online for brands of the company

Figure 15: Aggregate weekly brand exposure online for brands of the company and its competitors in 2011
Figure 16: Total online advertising exposures for category 1 and category 2
Figure 17: Histogram of number of umbrella brands households has been exposed to on TV

This graph shows among 4,831 households in the online dataset in the year 2010, what are the percentages of households who have been exposed to 1,2,3, ..., 35 umbrella brands; and among 5,747 households in the online dataset in the year 2011, what are the percentage of households who have been exposed to 1,2,3,..., 34 umbrella brands.
Figure 18: Total TV advertising exposures for category 1 and category 2
Figure 19: Estimated utility parameters for the full model
Figure 20: Estimated elasticities for the full model
Figure 21: Estimated goodwill production parameters for the full model
Figure 22: Estimated decay parameters for the full model
Figure 23: Estimated exposure parameters for the full model
References


**URL:** http://www.journals.marketingpower.com/doi/abs/10.1509/jmr.09.0401


Appendix

Again, we use Markov-Chain Monte Carlo (MCMC) methods to estimate the model. The estimation procedure is as follows:

1. Start with initial values for $\lambda, \gamma, \theta, \delta, \Sigma, \eta, \zeta$

2. For each $i$: Draw $u_{ijt}|\{X_{ijft}, Y_{ijt}\forall t\}, \theta_i, \lambda_i, \eta$

3. For each $i$: Draw $\theta_i, \delta_i, \lambda_i|\{u_{ijt}, A_{ijft}\forall j, f, t\}, \zeta, \eta$

4. For each $j, f$: Draw $\zeta_{jf}|\{A_{ijft}\forall i, t\}, \delta_i, \eta_j$

5. For each $j, t$: Draw $\eta_{jt}|\{u_{ijt}, A_{ijft}\forall i, f\}, \zeta$

6. Draw $\bar{\theta}|\theta, \Sigma$

7. Draw $\Sigma|\bar{\theta}, \theta$

We assume that the individual-level parameters are drawn from a normal distribution: $\theta_i \sim N(\bar{\theta}, \Sigma)$. We further assume normal priors for $\bar{\theta}, \zeta, \eta$, and the inverse normal cdf of $\lambda$. We assume inverse Wishart priors for $\Sigma$ and the variance of $\bar{\theta}, \zeta, \eta$, and the inverse normal cdf of $\lambda$.

We can write the conditional likelihood functions as follows:
\[ L_i(\theta_i\{u_{ijt}, A_{ijft}\forall j, f, t\}, \zeta, \eta) = \prod_i \prod_j \Phi(u_{ijt}) Y_{ijt} \prod_f \phi_{ijft} A_{ijf}^{ijf} \exp(-\phi_{ijft}) f(\theta_i|\bar{\theta}, \Sigma) \] (14)

\[ L_{jj}(\zeta_{jj}\{A_{ijft}\forall i, t\}, \delta_i, \eta_j) = \prod_i \prod_i \phi_{ijft} A_{ijf}^{ijf} \exp(-\phi_{ijft}) A_{ijf}^{ijf} \exp(-\phi_{ijft}) f(\zeta_{jj}|\bar{\zeta}, V_{\zeta}) \] (15)

\[ L_{jt}(\eta_{jt}\{u_{ijt}, A_{ijft}\forall i, f\}, \zeta) = \prod_i \Phi(u_{ijt}) Y_{ijt} \prod_f \phi_{ijft} A_{ijf}^{ijf} \exp(-\phi_{ijft}) f(\eta|\bar{\eta}, V_{\eta}) \] (16)

\[ L_{jt}(\bar{\theta}\{\theta_i\forall i\}, \Sigma) = f(\bar{\theta}|\theta_i\forall i, \Sigma) f(\bar{\theta}|\bar{\theta}, V_{\bar{\theta}}) \] (17)

\[ L_{jt}(\Sigma\{\theta_i\forall i\}, \bar{\theta}) = f(\Sigma|\theta_i\forall i, \bar{\theta}) f(\Sigma|a_{\Sigma}, b_{\Sigma}) \] (18)